U. C. BERKELEY - SPACE SCIENCES LAB

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HOW TO EAVESDROP ON ALIEN CHAT

Claudio Maccone

Member of the International Academy of Astronautics

E-mail: clmaccon@libero.it

Home page: http://www.maccone.com/

How to Eavesdrop on Alien Chat Summary - 1/3

ET, phone... each other? If aliens really are conversing, w are not picking up what they are saying. Now on researcher claims to have a way of tuning in to alie cellphone chatter.

On Earth, the signal used to send information vi cellphones has evolved from a single carrier wave to "spread spectrum" method of transmission. It's mor efficient, because chunks of information are essentiall carried on multiple low-powered carrier waves, and mor secure because the waves continually change frequenc so the signal is harder to intercept.

It follows that an advanced alien civilization would hav made this change too, but the search for extraterrestria life (SETI) is not listening for such signals, says Claudi Maccone, co-chair of the IAA SETI Permanent Study Grou based in Paris, France.

How to Eavesdrop on Alien Chat
Summary - 2/3

An algorithm known as the Fast Fourier Transform (FFT) i the method of choice for extracting an alien signal fron cosmic background noise. However, the technique cannc extract a spread spectrum signal. Maccone argues tha SETI should use an algorithm known as the Karhunen Loève Transform (KLT), which could find a buriec conversation with a signal-to-noise ratio 1000 times lowe than the FFT.

A few people have been "preaching the KLT" since th early 1980s but until now it has been impractical as i involves computing millions of simultaneous equations something even today's supercomputers would struggl with.

How to Eavesdrop on Alien Chat Summary - 3/3

At a recent meeting in Paris called “Searching for Lif Signatures”, Maccone presented a mathematical method t get around this burden and suggested that the KLT shoul be programmed into computers at the new Low Frequenc Array telescope in the Netherlands and the Squar Kilometer Array telescope, due for completion in 2012.

Seth Shostak at the SETI Institute in California agrees tha the KLT might be the way to go but thinks we shouldn abandon existing efforts yet. "It is likely that for their ow conversation they use a spread-spectrum method but it i not terribly crazy to assume that to get our attention the might use a 'ping' signal that has a lot of energy in narrow band - the kind of thing the FFT could find."

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The KLT (Karhunen-Loève Transform) to extend SETI searches to BROAD-BAND and EXTREMELY FEEBLE SIGNALS

Claudio Maccone

Member of the International Academy of Astronautics

E-mail: clmaccon@libero.it

Home page: http://www.maccone.com/

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Beyond the FFT : Extraction of Weak Signals from Noise plus Data Compression by the KLT.

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Member of the International Academy of Astronautics

E-mail: clmaccon@libero.it

Home page: http://www.maccone.com/

What is the KLT ?

Example (a Newtonian analogy):
onsider a solid object, like a BOOK,
escribed by its INERTIA MATRIX.

Then there exist only one special
eference frame where the Inertia
Matrix is DIAGONAL. This is the
eference frame spanned by the
IGENVECTORS of the Inertia
natrix.

$$I' = \begin{pmatrix} I'_{xx} & 0 & 0 \\ 0 & I'_{yy} & 0 \\ 0 & 0 & I'_{zz} \end{pmatrix}$$

KLT mathematics

- If $X(t)$ is a stochastic process (= input to the radio telescope) it can be expanded into an infinite series

$$X(t) = \sum_{n=1}^{\infty} Z_n \, \phi_n(t) \qquad 0 \le t \le T$$

- then
  - $\phi_n(t)$ are orthonormalized functions of the time:

$$\int_0^T \phi_m(t)\, \phi_n(t)\, dt = \delta_{mn}$$

  Z_n are random variables, not changing in time, with the property

$$\langle Z_m \, Z_n \rangle = \lambda_n \, \delta_{mn}$$

- In conclusion, the KLT separates the radiotelescope input (= noise + signal(s)) into ***UNCORRELATED*** components.

KLT mathematics

$$\int_0^T \langle X(t_1)\, X(t_2) \rangle \;\phi_n(t_1)\; dt_1 = \lambda_n\, \phi_n(t_2)$$

This is the integral equation yielding the Karhunen-Loève's eigenfunctions $\phi_n(t)$ and corresponding eigenvalues λ_n .

- The kernel of this integral equation is the autocorrelation.

- This is the best basis in the Hilbert space describing the signal+noise). The KLT adapts itself to the shape of the adiotelescope input (signal+noise) by adopting, as a reference rame, the one spanned by the eigenfunctions of the utocorrelation. And this is turns out to be just a LINEAR ransformation of coordinates in the Hilbert space. Thus, the KLT is an easily INVERTIBLE TRANSFORMATION

KLT filtering

• There is no degeneracy (i.e. each eigenvalue corresponds to just one eigenfuction only).

• The eigenvalues turn out to be the variances of the random variables Z_n, that is $\sigma^2_{Z_n} = \lambda_n = \left\langle Z_n^2 \right\rangle > 0.$

• Since $\left\langle Z_n \right\rangle = 0$ we can SORT in descending order of magnitude both the eigenvalues and the corresponding eigenfunctions. Then, if we decide to consider only the first few eigenfunctions as the "bulk" of the signal, and to apply the inverse KLT, that's what KLT filtering is: we just declare the taken-off part as "noise"!

• The Galileo mission by NASA-JPL used the KLT…

“Classic” KLT vs. FFT

	KLT	FFT	
↑	Works well for both wide and narrow band signals	Rigorously true for narrow band signals only	↓
↑	Works for both stationary and non-stationary input stochastic processes	Works OK for stationary input stochastic processes only	↓
↑	Is defined for any finite time interval	Is plagued by the “windowing” problems	↓
↓	Needs high computational burden: no “fast” KLT	Fast algorithm FFT	↑

NO Classic KLT in the 1990s

- If N is the size of the autocorrelation matrix (N may equal millions ore more in real SETI), the number of calculations requested to find the KLT is of the order of N square, while the same number for the FFT is much less: just N ln(N).

- This ***COMPUTATIONAL BURDEN*** prevented all SETI scientists from replacing the FFT by the KLT until 2007:

1) François Biraud et al. at Nançay in the years after 1983.

2) Bob Dixon et al. the Ohio State SETI Program after 1985.

3) Stelio Montebugnoli et al. at Medicina, Italy, after 1990.

BAM (=Bordered Autocorrelation Method) to EASILY find the KLT

- BAM is acronym for "BORDERED Autocorrelation Method".

- The new key idea is to regard the autocorrelation as a NEW FUNCTION OF KLT FINAL INSTANT, T. That is, to add one more row and one more column (= bordering) to the autocorrelation matrix for each new positive, increasing T.

- For STATIONARY processes, this amounts to the matrix:

$$R_{Toeplitz} = \begin{bmatrix} R_{XX}(0) & R_{XX}(1) & R_{XX}(2) & \cdots & \cdots & R_{XX}(N) \\ R_{XX}(1) & R_{XX}(0) & R_{XX}(1) & \cdots & \cdots & R_{XX}(N-1) \\ R_{XX}(2) & R_{XX}(1) & R_{XX}(0) & \cdots & \cdots & R_{XX}(N-2) \\ \cdots & \cdots & \cdots & R_{XX}(0) & \cdots & \cdots \\ R_{XX}(N) & R_{XX}(N-1) & \cdots & \cdots & R_{XX}(1) & R_{XX}(0) \end{bmatrix}$$

2007 Breakthrough in the KLT

- In the winter of 2006-7, the SETI-Italia Group at Medicina, (Stelio Montebugnoli, Francesco Schillirò, Salvo Pluchino and Claudio Maccone) discovered a way to CIRCUMVENT that big obstacle of the KLT N^2 computational burden.

- The idea is to use the BAM to exploit the **dependence on the final instant** T in both sides of the relationship, firstly proved in 1994 by Maccone in his KLT book, p.12, eq. (1.13):

$$\sum_{n=1}^{\infty} \lambda_n(T) = \int_0^T \sigma_{X(t)}^2 \, dt$$

BAM (Bordered Autocorrelation Method) to EASILY find the KLT

- Differentiating both sides wrt T yields the FINAL VARIANCE THEOREM (Maccone, Proc. of Science, 2007)

$$\sum_{n=1}^{\infty} \frac{\partial \lambda_n(T)}{\partial T} = \sigma^2_{X(T)}$$

- Confining ourselves to the FIRST EIGENVALUE (the "dominant one" = the largest) & STATIONARY ***X(t)*** ONLY:

$$\sum_{n=1}^{\infty} \frac{\partial \lambda_n(T)}{\partial T} \approx \frac{\partial \lambda_1(T)}{\partial T} = \sigma^2_X = \text{a\_CONSTANT\_wrt\_T}$$

- The SETI-Italia Team discovered that ***the Fourier transform of this constant is a peak (i.e. Dirac delta function) that is the FREQUENCY of the ET-SIGNAL !!!*** *As we now prove…*

BAM (Bordered Autocorrelation Method) to EASILY find the KLT

- Consider a pure tone buried into unitary White Noise *W*(*t*):

$$S(t) = W(t) + a\sin(\omega t) \quad (\mu << 1) \quad (0 \le t \le T)$$

- Write down the autocorrelation of this stationary process *S*(
assuming that the sinusoid and white noise are uncorrelated:

$$\langle S(t_1)\,S(t_2)\rangle = \langle W(t_1)\,W(t_2)\rangle + a^2 \sin(\omega t_1)\sin(\omega t_2)$$

- Now go over to the KLT of these two stationary processes:

$$S(t) = \sum_{n=1}^{\infty} rvS_n\, S_n(t) \quad W(t) = \sum_{n=1}^{\infty} rvW_n\, W_n(t)$$

BAM (Bordered Autocorrelation Method) to EASILY find the KLT

- The KLT expansion of both autocorrelations yields:

$$\sum_{n=1}^{\infty} \lambda_{Sn}\, S_n(t_1)\, S_n(t_2) = \sum_{n=1}^{\infty} \lambda_{Wn}\, W_n(t_1)\, W_n(t_2) + a^2 \sin(\omega\, t_1)\, \sin(\omega\, t_2$$

- Where we introduced the KLT eigenvalues of both processes.
- In addition, the White Noise autocorrelation simply is the Dirac's delta function, and so the White Noise KLT eigenfunctions may be ANY basis of orthonormalized time functions in the Hilbert space. For instance, they can be the simplest possible basis… the classical Fourier basis…

$$W_n(t) \equiv \sqrt{\frac{2}{T}}\, \sin\left(n\, \frac{2\pi}{T}\, t\right) \quad (n = 1, 2, \ldots) \quad (0 \le t \le T)$$

BAM (Bordered Autocorrelation Method) to EASILY find the KLT

- Replacing the last basis into the two KLT expansions abov yields a new form of these KLT expansions from which w can easily let both t sub 1 and t sub 2 DISAPPEAR by tw simple integrations over from zero to T. We skip these steps..
- The result after the two integrations is the RELATIONSHI AMONG THE EIGENVALUES

$$\lambda_{Sn} = \lambda_{Wn} + \frac{8\,\pi^2\,a^2\,n^2\,T}{\left(\omega^2\,T^2 - 4\,\pi^2\,n^2\right)^2}\sin^2(\omega\,T)$$

- This is the MOST IMPORTANT RESULT in the BAM-KLT !

BAM (Bordered Autocorrelation Method) to EASILY find the KLT

- It shows that for increasing *n* the two eigenvalues coincide.
- It shows that for increasing *T* the two eigenvalues coincide.
- But it also shows that the PERIODIC PART of the fractio goes like the sine SQUARE of omega, i.e. with a period c TWICE omega, i.e. TWICE the ET-FREQUENCY !!!

$$\lambda_{Sn} = \lambda_{Wn} + \frac{8\,\pi^2\,a^2\,n^2\,T}{\left(\omega^2\,T^2 - 4\,\pi^2\,n^2\right)^2} \cdot \left[\frac{1}{2} - \frac{1}{2}\cos(2\,\omega\,T)\right]$$

- ***This is the MOST IMPORTANT RESULT in the KLT ever !!!***

BAM (Bordered Autocorrelation Method) to EASILY find the KLT

- In other words still, we have found that the KLT eigenvalues of the PURE SIGNAL as a function of *T* are given by:

$$\lambda_n(T) = \frac{8\,\pi^2\,a^2\,n^2\,T}{\left(\omega^2\,T^2 - 4\,\pi^2\,n^2\right)^2} \cdot \left[\frac{1}{2} - \frac{1}{2}\cos(2\,\omega\,T)\right].$$

- What if we take the partial derivative of this wrt *T* ?
- Well, since *T* appears in three different places, the partial derivative will be the SUM of THREE terms:
- 1) a term in sin (2 omega T)
- 2) a term in cos (2 omega T)
- 3) a non-periodic term in T

BAM (Bordered Autocorrelation Method) to EASILY find the KLT

- Isn't this the FOURIER SERIES of the above partial derivative Of course it is! So we Fourier-invert that. And so...

- And so we have proved our KEY RESULT:
- **The ET-FREQUENCY equals half of the frequency of the inverse Fourier transform of the derivative wrt to T of the dominant KLT eigenvalue.**
- **This is the BAM-KLT METHOD, that allows for the ET frequency even for SNR very very low.**
- **SNR may be even 10 to the minus 5 or worse !!!**

SNR for BAM-KLT - 1/5

- For zero-mean-value (=centered) White Noise

$$\lambda_{Wn} = \left\langle Z_{Wn}^2 \right\rangle$$

- SNR definition for ET sinusoid in White Noise

$$SNR = \frac{power\_of\_the\_signal}{power\_of\_the\_noise} = \frac{a^2}{\left\langle Z_{Wn}^2 \right\rangle} = \frac{a^2}{\lambda_{Wn}}.$$

SNR for BAM-KLT - 2/5

- Re-write the relationship among eigenvalues

$$\lambda_{Sn}(T) = \lambda_{Wn} + \frac{8\,\pi^2\,a^2\,n^2\,T\,\sin^2(\omega\,T)}{\left(\omega^2\,T^2 - 4\,\pi^2\,n^2\right)^2}$$

upon dividing both sides by the noise eigenvalues and then rearranging

$$\frac{\lambda_{Sn}(T)}{\lambda_{Wn}} = 1 + \frac{a^2}{\lambda_{Wn}} \cdot \frac{8\,\pi^2\,n^2\,T\,\sin^2(\omega\,T)}{\left(\omega^2\,T^2 - 4\,\pi^2\,n^2\right)^2}$$

SNR for BAM-KLT - 3/5

- The SNR definition changes this into

$$\frac{\lambda_{Sn}(T)}{\lambda_{Wn}} = 1 + SNR \cdot \frac{8\,\pi^2\,n^2\,T\,\sin^2(\omega\,T)}{\left(\omega^2\,T^2 - 4\,\pi^2\,n^2\right)^2}$$

- Solving for SNR, this yields

$$SNR(T,n,\omega) = \left(\frac{\lambda_{Sn}(T)}{\lambda_{Wn}} - 1\right) \cdot \frac{\left(\omega^2\,T^2 - 4\,\pi^2\,n^2\right)^2}{8\,\pi^2\,n^2\,T\,\sin^2(\omega\,T)}.$$

SNR for BAM-KLT - 4/5

- The first factor on the right-hand side is SLIGHTLY POSITIVE if an ET signal is embedded into the white noise. But it equals ZERO if no ET signal is there:

$$\left(\frac{\lambda_{Sn}(T)}{\lambda_{Wn}} - 1\right) \geq 0$$

- The first KLT eigenvalue, n=1, is usually much larger then the other ones (**dominant eigenvalue**). Thus, we may simplify things by setting n=1 into the SNR :

SNR for BAM-KLT - 5/5

$$SNR(T,1,\omega)=\left(\frac{\lambda_{S1}(T)}{\lambda_{W1}}-1\right)\cdot\frac{\left(\omega^2\,T^2-4\,\pi^2\right)^2}{8\,\pi^2\,T\,\sin^2(\omega\,T)}.$$

- Letting $T\rightarrow\infty$, we see that SNR(T,1,ω) GROWS LIKE T CUBE. Thus, INCREASING *T* is the way to INCREASE the SNR to any desired value !!

The Next Steps…

This is as far as we could get !

Now it's up to YOU GUYS to :

1) **Write down the BAM-KLT computer CODE**

2) **Test it on large radiotelescopes**

3) **Search for BROAD-BAND Alien Chat !**

Thanks very much !