A NEW WAY OF PLANNING SETI SEARCHES BY THE STATISTICAL DRAKE EQUATION

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ABSTRACT

- We provide the statistical generalization of the Drak equation.
- From a simple product of seven positive numbers, th Drake equation is now turned into the product o seven positive random variables.
- We call this "the Statistical Drake Equation".
- A key advantage is to associate an ERROR BAR t each variable in the classical Drake equation.

The Classical Drake Equation /1

- In 1961 Frank Drake introduced his famous "Drake equation" described at the web site http://en.wikipedia.org/wiki/Drake_equation. It yields the number *N* of communicating civilizations in the Galaxy:

$$N = Ns \cdot fp \cdot ne \cdot fl \cdot fi \cdot fc \cdot fL$$

- Frank Donald Drake (b. 1930)

The Classical Drake Equation /2

- All Attendees in this meeting do know the meaning of the seven factors in the Drake equation: so, they will not be described here.
- On the contrary, we'd like to stress that the seven factors are just POSITIVE NUMBERS. And that the Drake equation simply is the PRODUCT of these seven positive numbers.
- It is claimed here that Drake's approach is too "simple-minded", since it does NOT yield the ERROR BAR associated to each factor!

The STATISTICAL Drake Equation /1

- If we want to associate an ERROR BAR to each factor of the Drake equation then…
- … we must regard each factor in the Drake equation as a RANDOM VARIABLE.
- Then the number *N* of communicating civilizations also becomes a random variable.
- This we call the STATISTICAL DRAKE EQUATION and study for the first time here.

The STATISTICAL Drake Equation /2

- Denoting each random variable by capitals, the STATISTICAL DRAKE EQUATION reads

$$N = \prod_{i=1}^{7} D_i$$

- Where the D sub i ("D from Drake") are the 7 random variables, and *N* is a random variable too ("to be determined").

The STATISTICAL Drake Equation /3

- The next question is: what is the statistical distribution of each *D sub i* ?
- To simplify things, we'll just suppose that each *D sub i* is UNIFORMLY DISTRIBUTED.
- Thus, the MEAN VALUE of each *D sub i* has same numeric value as the corresponding pure number in the Drake equation !
- Each *D sub i* also has an ERROR BAR around the mean value that is equal to the standard deviation of the UNIFORM distribution !

The STATISTICAL Drake Equation /4

- For instance: nobody knows what the number of stars in the Galaxy is *exactly !* ☺

- But in our Statistical Drake Equation, we avoid this rough approximation by replacing the unknown number of stars by a UNIFORM distribution centered around the most probable number. Also, it spreads out to the value of the standard deviation of the uniform distribution.

- It can be proven that the latter must be multiplied by a factor of sqrt(3) = 1.7.

The STATISTICAL Drake Equation /5

- This author has completely solved the problem of finding the statistical distribution of *N* given the 7 uniformly distributed random variables *D sub i*. It does not have a name and is rather complicated.
- The mathematical proof is too long to be described here in detail (sorry). It will be published soon.
- Let us further try to GENERALIZE the Statistical Drake Equation to the case where the random variables are OTHER THAN SEVEN. Then…

Generalizing the STATISTICAL Drake Equation to ANY NUMBER OF FACTORS /1

- Consider the statistical equation

$$N = \prod_{i=1}^{any\_number} D_i$$

- This is the generalization of our Statistical Drake Equation to the product of ANY finite NUMBER of positive random variables.

- Is it possible to determine the statistics of *N* ?

- Rather surprisingly, the answer is "yes" !

Generalizing the STATISTICAL Drake Equation to ANY NUMBER OF FACTORS /2

- First, you obviously take the natural log of both sides to change the finite product into a finite sum

$$\ln(N) = \sum_{i=1}^{any\_number} \ln(D_i)$$

- Second, to this finite sum one can apply the CENTRAL LIMIT THEOREM OF STATISTICS. It states that, in the limit for an infinite sum, the distribution of the left-hand-side is NORMAL.

- This is true WHATEVER the distributions of the random variables in the sum MAY BE. *Just Great !*

Generalizing the STATISTICAL Drake Equation to ANY NUMBER OF FACTORS /3

- So, the random variable on the left is NORMAL, i.e.

$$\ln(N)$$

- Thus, the random variable *N* under the log must be LOG-NORMAL and its distribution is determined!

- One must, however, determine the mean value and variance of this log-normal distribution in terms of the mean values and variances of the factor random variables. This is DIFFICULT. But it can be done, for example, by a suitable numeric code that this author wrote in MathCad language.

Conclusion #1
The number of Signaling Civilizations is Log-Normally distributed

- Our Statistical Drake Equation, now Generalized to any number of factors, embodies as a special case the Statistical Drake Equation with just 7 factors.

- The conclusion is that the random variable *N* (the number of communicating ET Civilizations in the Galaxy) is LOG-NORMALLY distributed.

- The classical "old pure-number Drake value" of *N* is now replaced by the MEAN VALUE of such a log-normal distribution.

- But we now also have an ERROR BAR around it !

Example #1
UNIFORM INPUT DISTRIBUTIONS

$Ns := 350 \cdot 10^{9}$ $\mu Ns := Ns$ $\sigma Ns := 1 \cdot 10^{9}$

$fp := \frac{50}{100}$ $\mu fp := fp$ $\sigma fp := \frac{10}{100}$

$ne := 1$ $\mu ne := ne$ $\sigma ne := \frac{1}{\sqrt{3}}$

$fl := \frac{50}{100}$ $\mu fl := fl$ $\sigma fl := \frac{10}{100}$

$fi := \frac{20}{100}$ $\mu fi := fi$ $\sigma fi := \frac{10}{100}$

$fc := \frac{20}{100}$ $\mu fc := fc$ $\sigma fc := \frac{10}{100}$

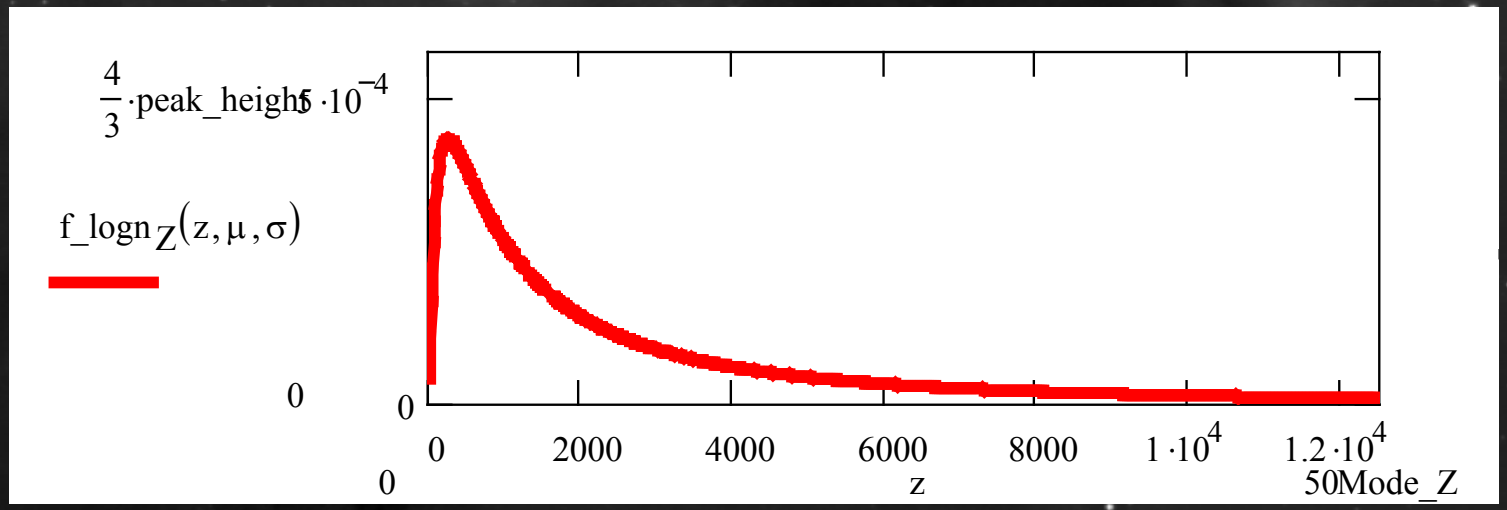
$fL := \frac{10000}{10^{10}}$ $\mu fL := fL$ $\sigma fL := \frac{1000}{10^{10}}$

$N := Ns \cdot fp \cdot ne \cdot fl \cdot fi \cdot fc \cdot fL$ $fL\_factor := Ns \cdot fp \cdot ne \cdot fl \cdot fi \cdot fc$ $\mu fL = 0.000001$

$N = 3500$ $fL\_factor = 3500000000$ $\sigma fL = 0.0000001$

Example #1
The number of Signaling Civilizations is Log-Normally distributed

- For the above uniform inputs, the log-normal curve



- Yields the number of Signaling Civilizations (mean value) as 4590, while classic Drake eq. yields 3500.

- With a standard deviation of ± 11,195 Civilizations.

Conclusion #2

The DISTANCE between ET Civilizations now follows a probability distribution

- In all Astrobiology textbooks (e.g. Bennett & Shostak 2007 ed., page 404) the average DISTANCE betwee Signaling Civilizations in the Galaxy is given. This i inversely proportional to the cubic root of the numbe N obtained by the classical Drake equation.

- Now this average distance is a new random variable This author found the relevant probability distribution But this is too complicated to be described here.

- The MEAN VALUE of such a new distribution is the ol average DISTANCE, but we now also have an ERRO BAR around it !

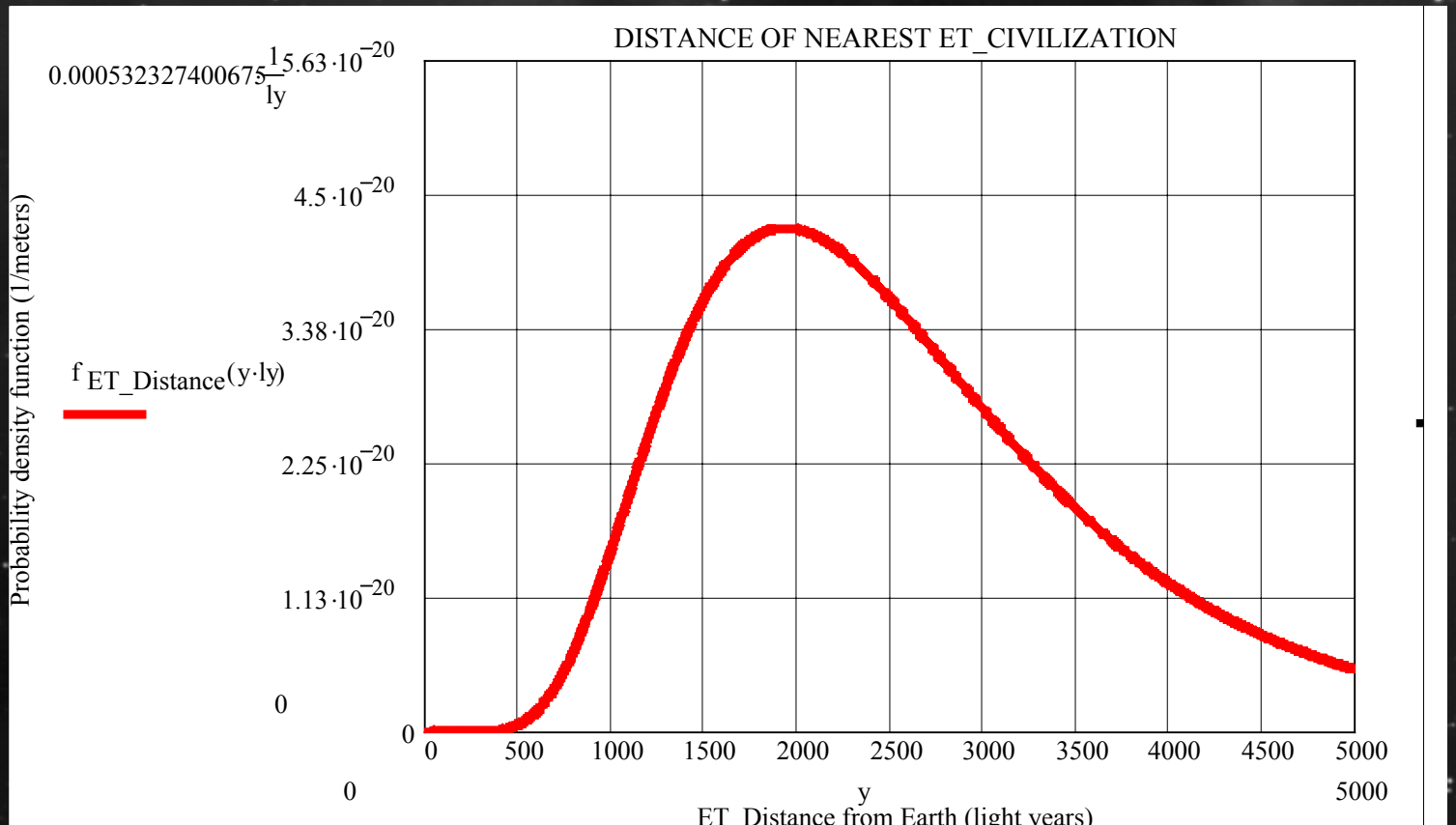
Example #2

The DISTANCE between ET Civilizations now follows a probability distribution

- In the previous example, the average DISTANCE given by the classic Drake equation is 4093 light years.
- In our statistical approach, the MEAN VALUE of the distance turns out to be 2670 light years...
- ...with an ERROR BAR of ± 1309 light years.
- Thus the MINIMAL STATISTICAL ET DISTANCE is JUST 1361 light years. This is *SETI-ENCOURAGING !*

Example #2

The DISTANCE between ET Civilizations now follows a probability distribution



CONCLUSION

- Surprisingly, STATISTICS and the Drake equation MATCH PERFECTLY if one just takes the logs of the Drake equation!
- By doing so, one also invokes the Central Limit Theorem (CLT) of Statistics, enabling ANY number of factors to be considered.
- This is our DATA ENRICHMENT PRINCIPLE that is the key to further future Astrobiology investigations by virtue of our extension.


Thanks very much !